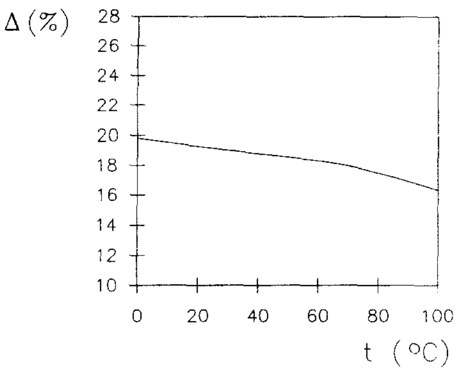

FIG. 3. Error vs temperature.

developed for calculating the Nusselt number on the wall. This formula includes both the effects of the test section configuration and the effects of the ratio of the index of refraction of the test material to the index of refraction of the materials outside the test cell. The relative error can be estimated by equation (24). Generally, the index of refraction of the optical medium in the test section often varies with the temperature. Thus, the error induced also depends on the temperature, when the test section configuration and measuring system are determined. Therefore, the use of equation (23) is suggested, rather than equation (9), when the shadowgraph method is used quantitatively. Although five layers of the medium outside of the test cell are taken into account in this paper, equation (23) can be applied to other cases with any number of different optical materials outside the test section; for example, m layers. In such cases, the upper limit of summation in equations (23) and (24) need to be changed to m instead of 5.

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REFERENCES

1. R. J. Goldstein, Optical techniques for temperature measurements. In *Measurements in Heat Transfer*. Hemisphere, Washington, DC (1976).
2. R. J. Goldstein, Optical systems for flow measurement: shadowgraph, Schlieren, and interferometric techniques. In *Fluid Mechanics Measurements*. Hemisphere, Washington, DC (1983).
3. W. Hauf and U. Grigull, Optical method in heat transfer. In *Advances in Heat Transfer*, Vol. 6. Academic Press, New York (1970).
4. M. S. Uberoi and L. S. G. Kovasznay, Analysis of turbulent density fluctuations by the shadow method, *J. Appl. Phys.* **26**, 19–24 (1955).
5. M. R. Davis, Measurements in a subsonic jet using a quantitative Schlieren technique, *J. Fluid Mech.* **46**, 631–656 (1971).
6. L. Hesselink and B. S. White, Digital image processing of flow visualized photographs, *Appl. Opt.* **22**, 1454–1461 (1983).
7. T. Grandke, Theory and application of the laser shadow technique, part 1. Two dimensional turbulent flow, *Exp. Fluids* **3**, 77–85 (1985).
8. T. Grandke, Theory and application of the laser shadow technique, part 2. Axisymmetric turbulent flow, *Exp. Fluids* **3**, 277–282 (1985).
9. R. D. White, A. G. Bathelt, W. Leindenfrost and R. Viskanta, Study of heat transfer and melting from a cylinder embedded in a phase change material, ASME Paper 77-HT-42 (1977).
10. A. G. Bathelt and R. Viskanta, Heat transfer at the solid–liquid interface during melting from a horizontal cylinder, *Int. J. Heat Mass Transfer* **23**, 1493 (1980).
11. Q. J. Wang and W. Y. Li, Heat transfer and interface motion in the presence of subcooling for melting around a horizontal tube with and without axial fins, advance in phase change heat transfer, *Proc. Int. Symp. on Phase Change Heat Transfer* (Edited by M. D. Xin), p. 350 (1988).
12. Z. F. Dong, Research on solid–liquid phase change heat transfer in a rectangular cavity with phase change material—contact melting study, Ph.D. Thesis, Xi'an Jiaotong University, China (1988).

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An improved velocity field for the Madejski splat-quench solidification model

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RAPID solidification processing (RSP) consists of the production of materials from the melt, at high rates of cooling and freezing, in order to achieve certain desired or perhaps unusual characteristics. Many forms of RSP are currently in use, one of the most elementary being the 'splat-quenching' of liquid-metal drops by impact onto a solid substrate. A mathematical model of the splat-quench process was developed some time ago by Madejski [1, 2] and he obtained reasonably good agreement with experiment. In fact, his model is still finding use [3] as an aid in the interpretation of experimental splat-quenching data. In this note, we make a substantial improvement upon the velocity field used by Madejski and demonstrate that the model obtained therewith is correspondingly improved.

Madejski postulated a velocity field for the unsolidified portion of the liquid drop as it spreads on the substrate. He then required that the time derivative of the mechanical plus interfacial energy of the drop be zero. The velocity field that he assumed was extremely simple, and is expressed in cylindrical coordinates as

$$v_z = -Cz^2 \quad (1)$$

$$v_r = Czr \quad (2)$$

where v_r and v_z are, respectively, the radial and axial components of the velocity field, z is the coordinate measured perpendicular to the substrate (he called this coordinate x), r is the radial coordinate measured parallel to the substrate, and C is a time-dependent (but spatially independent) quantity which can be expressed in terms of a combination of parameters.

This velocity field satisfies the continuity relation for an incompressible fluid,

$$\nabla \cdot \mathbf{v} = 0, \quad (3)$$

and also the requirement that the velocity components be zero at the interface (taken as $z = 0$) between the melt and the solidified portion of the drop. However, this field does *not* satisfy the requirement that

$$\frac{\partial v_r}{\partial z}(z = b) = 0 \quad (4)$$

where the surface $z = b$ is taken as the upper surface of the spreading drop. The net effect is essentially that of a shear stress being artificially applied to the upper drop surface. The consequence of this fact can be demonstrated by considering the local volumetric rate of change of viscous energy, E_{vis}, which is given by

$$E_{vis} = \mathbf{v} \cdot \mu \nabla^2 \mathbf{v} \quad (5)$$

where μ is the viscosity. One can readily show that E_{vis} is equal to $2\mu C^2 z^2$ for this velocity field and thus is positive *everywhere* within the liquid, reflecting the fact that energy is effectively being transmitted to the spreading drop at its upper surface.

The following is a velocity field which, as we shall demonstrate, is an improvement over that given by equations (1) and (2):

$$v_z = 2D\left(\frac{z^3}{3} - bz^2\right) \quad (6)$$

$$v_r = Dr(2zb - z^2) \quad (7)$$

where D is a time-dependent quantity analogous to C in equations (1) and (2). Clearly, the continuity relation, equation (3), is still satisfied, as is the condition that both velocity components vanish at $z = 0$. However, this field also satisfies equation (4), so that the upper drop surface experiences no shear stress for this case.

Moreover, we can use equation (5) to show that for this velocity field, E_{vis} is given by

$$E_{vis} = -2\mu D^2 \left[(2zb - z^2)r^2 + 4\left(\frac{z^3}{3} - bz^2\right)(b - z)\right] \quad (8)$$

which can be shown to be negative everywhere within the drop except in a region around the axis of symmetry (the z-axis), the perimeter of which extends out no further, in the radial direction, than about

$$r \sim b \quad (9)$$

and hence involves a progressively smaller volume of liquid as the drop spreads and as solidification progresses. Integration of E_{vis} over the entire volume, V, of the unsolidified portion of the drop yields

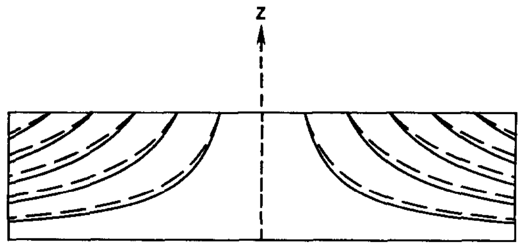

FIG. 1. Streamlines for the Madejski velocity field (dashed curves) and for the improved field (solid curves) within a cylindrical volume of liquid having rectangular cross-section, as shown. The lower and upper boundaries of this volume are the planes $z = 0$ and $z = b$, respectively, and the other boundary is the surface $r = R$. Flow is generally directed downward and away from the z-axis.

$$\int_V E_{vis}\, dV = -\frac{2\pi}{3}\mu D^2 R^2 b^3 \left(R^2 - \frac{4b^2}{5}\right) \quad (10)$$

where R is the instantaneous radius of the spreading drop. Clearly, this integral is negative as long as

$$R > 2b/\sqrt{5} \quad (11)$$

which generally is satisfied, since the radius of the spreading drop is usually large compared to its thickness. The fact that the integral *can* be negative (for small enough R) reflects the approximate nature of even the improved velocity field.

Streamlines for the two velocity fields discussed here can be easily determined. In particular, for the original Madejski model, the streamlines are given by

$$rz = \text{constant} \quad (12)$$

and for the improved velocity field they are

$$rz(3b - z)^{1/2} = \text{constant}. \quad (13)$$

Some examples for each field are shown in Fig. 1 for a cylindrical volume of liquid having a rectangular cross-section. In this figure, streamlines for both cases emanate from the same set of points along the upper surface of the rectangle. One difference between the two cases is that the ratio of z-component to r-component of velocity is larger in magnitude, at any given point, for the improved field than for the Madejski field. This means that transport of flowing liquid is directed more predominantly, for the new field, in a direction *toward* the advancing melt/solid interface rather than *parallel* to the interface. The consequence of this fact, in terms of overall effects on liquid cooling and solidification, can be assessed by applying the energy-balance approach used by Madejski.

REFERENCES

1. J. Madejski, Solidification of droplets on a cold surface, *Int. J. Heat Mass Transfer* **19**, 1009–1013 (1976).
2. J. Madejski, Droplets on impact with a solid surface, *Int. J. Heat Mass Transfer* **26**, 1095–1098 (1983).
3. E. W. Collings, A. J. Markworth, J. K. McCoy and J. H. Saunders, Splat-quench solidification of freely falling liquid-metal drops by impact on a planar substrate, *J. Mater. Sci.* **25**, 3677–3682 (1990).